$$P_{c2a} = 4k\frac{2V^2}{\pi h}\int_0^{n\Delta u}\frac{dv}{\sqrt{[(\Delta u)^2 + v^2]}} = \frac{8V^2k}{\pi h}\log 2n \quad (20)$$

b) $$v = n\Delta u \text{ —— } v_p$$

$$P_{c2b} = 4k\frac{2V^2}{\pi h}\int_{n\Delta u}^{v_p}\frac{dv}{\tanh v}$$

$$= \frac{8V^2k}{\pi h}\left(\frac{\pi b}{4h} - 1 - \log n - \log \Delta u\right) \quad (21)$$

v_p given by (5).

The sum of (19), (20) and (21) is

$$P_c \cong \frac{4V^2k}{\pi h}\left(\frac{b}{h} + \frac{2}{\pi}\log\frac{4h}{\pi d}\right).$$

ACKNOWLEDGMENT

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Transmission Characteristics of Sandwiches*

The radome design equations which are in common use are based on the transmission characteristics of sandwiches. Transmission line techniques provide a convenient method for determining these characteristics. The transmission line analogy of wave propagation has been discussed by Ramo and Whinnery.[1] A restricted application of this analogy is given in a report by Snow.[2] In this note, a more general application will be discussed.

The sandwich shown in Fig. 1 will be considered. Although this sandwich has only three layers, this does not indicate that the procedure is limited to any maximum number of sheets. It is assumed that the sandwich consists of plane, homogeneous, isotropic sheets of infinite extent. The sheets may be lossy. (A lossy dielectric can be characterized by either the dielectric constant or permeability or both being complex.) The wave incident upon the sandwich is plane and linearly polarized.

Since the transmission characteristics are different when the polarization is perpendicular and parallel to the plane of incidence, it is necessary to consider these two polarizations separately. When the polarization of the wave is neither perpendicular nor parallel to the plane of incidence, it is necessary to divide the wave into perpendicular and parallel components. The subscripts

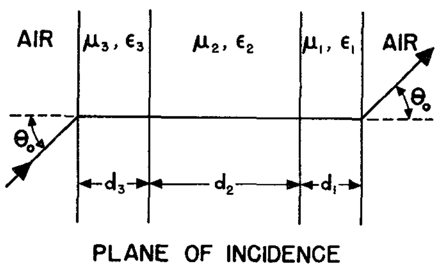

Fig. 1—A three-layer sandwich.

"⊥" and "‖" denote components and parameters associated with the components of the wave whose polarizations are perpendicular and parallel, respectively, to the plane of incidence.

The ith sheet is analogous to a section of transmission line whose length is d_i, whose propagation constant is

$$\gamma_i = \alpha_i + j\beta_i = j(2\pi/\lambda_0)\sqrt{\mu_i\epsilon_i - \sin^2\theta_0}, \quad (1)$$

and whose characteristic impedance is

$$\eta_{i\perp} = \mu_i/\sqrt{\mu_i\epsilon_i - \sin^2\theta_0} \quad (2)$$

or

$$\eta_{i\parallel} = \sqrt{\mu_i\epsilon_i - \sin^2\theta_0}/\epsilon_i, \quad (3)$$

where λ_0 is the wavelength in air. In general, $\eta_{i\perp}$ and $\eta_{i\parallel}$ are complex. The incident electric field E_4^+ required to produce a unit field at the opposite side of the sandwich is given by the equation

$$E_4^+ = \left(\frac{1+\Gamma_0'}{1+\Gamma_1}\right)\left(\frac{1+\Gamma_1'}{1+\Gamma_2}\right)\left(\frac{1+\Gamma_2'}{1+\Gamma_3}\right)\cdot\left(\frac{1+\Gamma_3'}{1+\Gamma_4}\right)e^{\gamma_1 d_1+\gamma_2 d_2+\gamma_3 d_3}, \quad (4)$$

where

$$\Gamma_0' = 0, \quad (5)$$

$$\Gamma_i' = \Gamma_i e^{-2\gamma_i d_i}, \quad (6)$$

$$\Gamma_{i+1} = \frac{\eta_i(1+\Gamma_i') - \eta_{i+1}(1-\Gamma_i')}{\eta_i(1+\Gamma_i') + \eta_{i+1}(1-\Gamma_i')}. \quad (7)$$

Transmission line charts, i.e., Smith $(R-X)$ and Carter $(Z-\theta)$ charts, provide a convenient method for determining E_4^+.

The voltage reflection coefficient is Γ_4 and the power reflection coefficient is $|\Gamma_4|^2$. The ratio of power transmitted to power incident is $1/|E_4^+|^2$. The increase in phase retardation due to the sandwich is

$$\Phi = (\text{angle of } E_4^+) - (360°/\lambda_0)(d_1 + d_2 + d_3)\cos\theta_0. \quad (8)$$

* Presented at the Symposium on Antennas and Radomes in Tracking Systems, University of Vermont, Burlington, Vermont, June 9 and 10, 1955.

[1] Ramo and Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, New York, pp. 250–266; 1944.

[2] O. J. Snow, "Report on Applications of the Impedance Concept to Radome Wall Design," Aeronautical Electronic and Electrical Laboratory, U. S. Naval Air Dev. Center, Johnsville, Pa., Rep. No. NADC-EL-52196; April, 1953.

To illustrate how a $Z-\theta$ transmission line chart can be used to determine the transmission characteristics of a sandwich, an example will be considered. For this example, the polarization is perpendicular to the plane of incidence, $\lambda_0=3.2$ cm $=1.26$ inch, $\theta_0=45°$, $d_1=0.05$ inch, $\mu_1=1$, $\epsilon_1=6$, $d_2=0.21$ inch, $\mu_2=0.8(1+j0.05)$, $\epsilon_2=4(1-j0.2)$, $d_3=0.07$ inch, $\mu_3=1$, and $\epsilon_3=4$. Now $\eta_{0\perp}=\eta_{4\perp}=1.414$, $\eta_{1\perp}=0.426$, $\eta_{2\perp}=0.481\underline{/7.9°}$, $\eta_{3\perp}=0.535$, $\alpha_1=\alpha_3=0$, $\alpha_2=0.721$ nepers per inch, $\beta_1=(2\pi)(1.861)$ radians per inch, $\beta_2=(2\pi)\cdot(1.317)$ radians per inch, and $\beta_3=(2\pi)(1.485)$ radians per inch.

The point $Z_1=\eta_{0\perp}/\eta_{1\perp}=3.317$ is plotted on a Z-θ chart, as in Fig. 2 (below). The

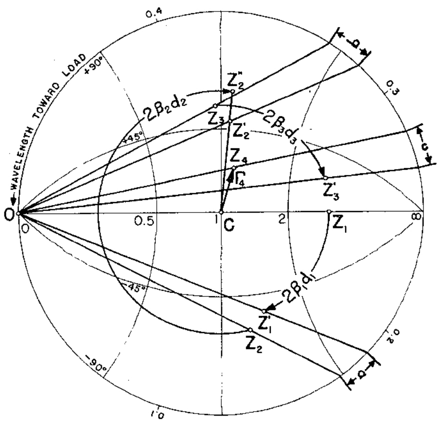

Fig. 2—Graphical construction for example.

point Z_1 is rotated through the angle $2\beta_1 d_1$ or $(1.861)(0.05)\lambda=0.093\lambda$ to obtain $Z_1'=1.40\underline{/-54°}$. The point $Z_2=(\eta_{1\perp}/\eta_{2\perp})Z_1'=1.24\underline{/-61.9°}$ is plotted. The point Z_2 is rotated through the angle $2\beta_2 d_2$ or $(1.317)\cdot(0.21)\lambda=0.278\lambda$ to obtain Z_2''. The distance $\overline{CZ_2''}e^{-2\alpha_2 d_2}=\overline{CZ_2''}e^{-0.0303}=0.739\overline{CZ_2''}$ is measured from C along the line CZ'' to locate the point $Z_2'=1.07\underline{/49°}$. The point $Z_3=(\eta_{2\perp}/\eta_{3\perp})Z_2'=0.96\underline{/56.9°}$ is plotted. The point Z_3 is rotated through the angle $2\beta_3 d_3$ or $(1.485)(0.07)\lambda=0.104\lambda$ to obtain $Z_3'=2.98\underline{/25°}$. The point $Z_4=(\eta_{3\perp}/\eta_{4\perp})Z_3'=1.13\underline{/25°}$ is plotted.

The magnitude of the voltage reflection coefficient is $\overline{CZ_4}$/(radius of chart) $=0.23$ Now

$$|E_4^+| = \frac{\overline{OC}}{\overline{OZ_1}}\,\frac{\overline{OZ_1'}}{\overline{OZ_2}}\,\frac{\overline{OZ_2'}}{\overline{OZ_3}}\,\frac{\overline{OZ_3'}}{\overline{OZ_4}}\,e^{\alpha_1 d_1+\alpha_2 d_2+\alpha_3 d_3} = 1.10$$

and the ratio of power transmitted to power incident is 0.826. The angle of E_4^+ is

$$\begin{aligned} a - b - c + \beta_1 d_1 + \beta_2 d_2 + \beta_3 d_3 &= 360°(0.014 - 0.016 - 0.016 + 0.093 \\ &\quad + 0.278 + 0.104) \\ &= 164.5°, \end{aligned}$$

and the increase in phase retardation due to the sandwich is

$$\Phi=164.5°-(360°/1.26)(0.33)\cos 45°=9.78°.$$

A graphical construction for determining the transmission characteristics of a sandwich has the advantage that the effect of the individual layers is presented in visual form. Ways to vary the parameters to obtain a desired result may be suggested by a study of the chart.

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Measurement of Reflection Coefficients through a Lossless Network

Often it is not convenient to measure a reflection coefficient directly. In some such cases, the following procedure may be used. The arrangement of components shown in Fig. 1 will be considered. It will be assumed that lines Nos. 1 and 2 are lossless. These lines may be transmission lines, waveguides, or one of each.

Fig. 1—Arrangement for measuring reflection coefficients.

First, a matched load is connected to line No. 2 and the matching network is adjusted until the reflection coefficient in line No. 1 is zero. Next, the matched load is replaced by a short circuit. The magnitude of the reflection coefficient in line No. 1 should be one. This should be checked experimentally. A voltage null in line No. 1 is located and designated the *short-circuit point.* Finally, the short circuit is replaced by the unknown impedance. The reflection coefficient $\mathbf{\Gamma}_1$ in line No. 1 is measured relative to the short-circuit point. The reflection coefficient $\mathbf{\Gamma}_2$ in line No. 2 related to the point where the short circuit was connected is the same as $\mathbf{\Gamma}_1$.

The theoretical basis for this procedure is the well-known equation

$$\mathbf{\Gamma}_1 = \frac{\boldsymbol{a}\mathbf{\Gamma}_2 + \boldsymbol{b}}{\boldsymbol{c}\mathbf{\Gamma}_2 + \boldsymbol{d}}.$$

Since $\mathbf{\Gamma}_1=0$ when $\mathbf{\Gamma}_2=0$, $\boldsymbol{b}=0$. Since the various components are lossless, $\Gamma_1=1$ when $\Gamma_2=1$. Consequently, $|\boldsymbol{c}\mathbf{\Gamma}_2+\boldsymbol{d}|$ is equal to a constant for all values of $\mathbf{\Gamma}_2$ such that $\Gamma_2=1$. This requires that $\boldsymbol{c}=0$. Now $\mathbf{\Gamma}_1=(\boldsymbol{a}/\boldsymbol{d})\mathbf{\Gamma}_2$, where $a/d=1$. When $\mathbf{\Gamma}_1$ is referred to the short-circuit point, $\boldsymbol{a}/\boldsymbol{d}=1$ and $\mathbf{\Gamma}_1=\mathbf{\Gamma}_2$.

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For a photograph and biography of S. B. Cohn, see the March, 1955 issue of the Transactions of the IRE-PGMTT.

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